Chp.4 : Consumption, Saving, and Investment	Practice Problems
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1. Julia is making saving plans for this year and next. She knows her real income after taxes will be \$50,000 in both years. Any part of her income saved this year will earn a real interest rate of 10% between this year and next year. Currently, the consumer has no wealth (no money in the bank or other financial assets, and no debts). There is no uncertainty about the future.

   The consumer wants to save an amount this year that will allow her to make college tuition payments next year equal to \$16,800 in real terms; enjoy exactly the same amount of consumption this year and next year, not counting tuition payments as part of next year's consumption; and have neither assets nor debts at the end of next year.

   (a) How much should the consumer save this year? How much should she consume?

   Solution: Given: $Y_1 = Y_2 = \$50,000$, $r = 10\%$, $T = \$16,800$
   Since Julia wants equal consumption in both periods ($C_1 = C_2 = C$), we establish the key equation:

   $$(Y_1 - C)(1 + r) + (Y_2 - C) = T \tag{1}$$
   $$(\$50,000 - C) \times 1.1 + (\$50,000 - C) = \$16,800 \tag{2}$$

   Factoring out $(\$50,000 - C)$:

   $$(\$50,000 - C)(1.1 + 1) = \$16,800 \tag{3}$$
   $$(\$50,000 - C) \times 2.1 = \$16,800 \tag{4}$$
   $$\$50,000 - C = \frac{\$16,800}{2.1} = \$8,000 \tag{5}$$

   Therefore:

   - Consumption: $C = \$50,000 - \$8,000 = \boxed{\$42,000}$
   - Savings: $S = Y_1 - C = \$50,000 - \$42,000 = \boxed{\$8,000}$

   Hint: Why does the multiplier become 2.1? Consider how savings compound and support consumption in both periods.

   (b) Her current income rises from \$50,000 to \$54,200.

Solution: New given: $Y_1 = \$54,200$
The key equation becomes:

$$(\$54,200 - C) \times 1.1 + (\$50,000 - C) = \$16,800 \tag{6}$$

Expanding:

$$\$54,200 \times 1.1 + \$50,000 - C \times 2.1 = \$16,800 \tag{7}$$
$$\$59,620 + \$50,000 - \$16,800 = C \times 2.1 \tag{8}$$
$$\$92,820 = C \times 2.1 \tag{9}$$
$$C = \frac{\$92,820}{2.1} = \$44,200 \tag{10}$$

Therefore:

- Consumption: $C = \boxed{\$44,200}$
- Savings: $S = \$54,200 - \$44,200 = \boxed{\$10,000}$

Economic Insight: A rise in current income increases saving.
Hint: How is the \$4,200 income increase distributed between consumption and savings?

(c) The income she expects to receive next year rises from \$50,000 to \$54,200.

Solution: New given: $Y_2 = \$54,200$
The key equation becomes:

$$(\$50,000 - C) \times 1.1 + (\$54,200 - C) = \$16,800 \tag{11}$$

Expanding:

$$\$50,000 \times 1.1 + \$54,200 - C \times 2.1 = \$16,800 \tag{12}$$
$$\$55,000 + \$54,200 - \$16,800 = C \times 2.1 \tag{13}$$
$$\$92,400 = C \times 2.1 \tag{14}$$
$$C = \frac{\$92,400}{2.1} = \$44,000 \tag{15}$$

Therefore:

- Consumption: $C = \boxed{\$44,000}$
- Savings: $S = \$50,000 - \$44,000 = \boxed{\$6,000}$

Economic Insight: A rise in future income decreases saving (consumption smoothing effect).
Hint: Compare with part (b). Why does future income increase lead to lower savings than current income increase?

(d) During the current year she receives an inheritance of \$1,050 (an increase in wealth, not income).

Solution: New given: Initial wealth $W = \$1,050$
With initial wealth, the budget constraint incorporates the wealth:

$$(W + Y_1 - C)(1 + r) + (Y_2 - C) = T \tag{16}$$
$$(\$1,050 + \$50,000 - C) \times 1.1 + (\$50,000 - C) = \$16,800 \tag{17}$$

Simplifying:

$$(\$51,050 - C) \times 1.1 + (\$50,000 - C) = \$16,800 \tag{18}$$
$$\$56,155 - C \times 1.1 + \$50,000 - C = \$16,800 \tag{19}$$
$$\$106,155 - C \times 2.1 = \$16,800 \tag{20}$$
$$C \times 2.1 = \$89,355 \tag{21}$$
$$C = \frac{\$89,355}{2.1} = \$42,550 \tag{22}$$

Therefore:

- Consumption: $C = \boxed{\$42,550}$
- Savings: $S = \$50,000 - \$42,550 = \boxed{\$7,450}$

Economic Insight: A rise in wealth decreases saving (wealth enables higher consumption while meeting future goals).
Hint: Why doesn't the entire \$1,050 inheritance go directly to savings?

(e) The expected tuition payment for next year rises from \$16,800 to \$18,900 in real terms.

Solution: New given: $T = \$18,900$
The key equation becomes:

$$(\$50,000 - C) \times 1.1 + (\$50,000 - C) = \$18,900 \tag{23}$$
$$(\$50,000 - C) \times 2.1 = \$18,900 \tag{24}$$
$$\$50,000 - C = \frac{\$18,900}{2.1} = \$9,000 \tag{25}$$

Therefore:

- Consumption: $C = \$50,000 - \$9,000 = \boxed{\$41,000}$
- Savings: $S = \$50,000 - \$41,000 = \boxed{\$9,000}$

Economic Insight: Higher future obligations require more current saving.
Hint: The tuition increased by \$2,100. How much did savings increase? What does this tell you about burden sharing?

(f) The real interest rate rises from 10% to 24%.

Solution: New given: $r = 24\%$ (or 0.24)
The key equation becomes:

$$(\$50,000 - C) \times 1.24 + (\$50,000 - C) = \$16,800 \tag{26}$$
$$(\$50,000 - C)(1.24 + 1) = \$16,800 \tag{27}$$
$$(\$50,000 - C) \times 2.24 = \$16,800 \tag{28}$$
$$\$50,000 - C = \frac{\$16,800}{2.24} = \$7,500 \tag{29}$$

Therefore:

- Consumption: $C = \$50,000 - \$7,500 = \boxed{\$42,500}$
- Savings: $S = \$50,000 - \$42,500 = \boxed{\$7,500}$

Economic Insight: With a fixed wealth target, higher interest rates reduce required saving (substitution effect dominates).
Hint: Why does a higher interest rate lead to LOWER savings when there's a fixed tuition target?

3. You have just taken a job that requires you to move to a new city. In relocating, you face the decision of whether to buy or rent a house. A suitable house costs $300,000 and you have saved enough for the down payment. The (nominal) mortgage interest rate is 10% per year, and you can also earn 10% per year on savings. Mortgage interest payments are tax deductible, interest earnings on savings are taxable, and you are in a 30% tax bracket. Interest is paid or received, and taxes are paid, on the last day of the year. The expected inflation rate is 5% per year.

   The cost of maintaining the house (replacing worn-out roofing, painting, and so on) is 6% of the value of the house. Assume that these expenses also are paid entirely on the last day of the year. If the maintenance is done, the house retains its full real value. There are no other relevant costs or expenses.

   1. What is the expected real after-tax interest rate on the home mortgage?
   2. What is the user cost of the house?
   3. If all you care about is minimizing your living expenses, at what (annual) rent level would you be just indifferent between buying a house and renting a house of comparable quality? Rent is also paid on the last day of the year.

(a) What is the expected real after-tax interest rate on the home mortgage?

Solution: To find the expected real after-tax interest rate, we need to:

1. Calculate the after-tax nominal interest rate
2. Adjust for inflation to get the real rate

Given:

- Nominal mortgage interest rate: $i = 10\%$ or 0.10
- Tax rate: $t = 30\%$ or 0.30
- Expected inflation rate: $\pi^e = 5\%$ or 0.05
- Mortgage interest is tax deductible

The after-tax nominal interest rate is:

$$\begin{aligned} i_{\text{after-tax}} &= i(1-t) && (30)\\ &= 0.10(1-0.30) && (31)\\ &= 0.10 \times 0.70 && (32)\\ &= 0.07 && (33) \end{aligned}$$

The expected real after-tax interest rate is:

$$\begin{aligned} r &= i(1-t) - \pi^e && (34)\\ &= 0.07 - 0.05 && (35)\\ &= \boxed{0.02 \text{ or } 2\%} && (36) \end{aligned}$$

Hint: Why do we subtract the inflation rate? Think about how inflation affects the real burden of debt repayment.

(b) What is the user cost of the house?

Solution: The user cost of capital for housing includes:

1. The opportunity cost of capital (real after-tax interest rate)
2. Depreciation/maintenance costs

Given:

- House price: $p_K = \$300,000$
- Real after-tax interest rate (from part a): $r = 0.02$
- Depreciation/maintenance rate: $d = 6\%$ or 0.06

The annual user cost of capital formula is:

$$\begin{aligned} uc &= (r+d) \times p_K && (37)\\ &= (0.02 + 0.06) \times \$300,000 && (38)\\ &= 0.08 \times \$300,000 && (39)\\ &= \boxed{\$24,000} && (40) \end{aligned}$$

Economic Interpretation: The \$24,000 represents:

- \$6,000 in opportunity cost (2% of \$300,000)
- \$18,000 in maintenance/depreciation (6% of \$300,000)

Hint: The user cost represents the true economic cost of owning the house for one year, including both financial and physical costs.

(c) If all you care about is minimizing your living expenses, at what (annual) rent level would you be just indifferent between buying a house and renting a house of comparable quality? Rent is also paid on the last day of the year.

Solution: To find the indifference rent level, we need to compare the costs of owning versus renting.

Cost of Owning: From part (b), the annual user cost of owning the house is \$24,000.

Cost of Renting: The annual rent payment is R (to be determined).

Indifference Condition: You are indifferent between buying and renting when:

$$\text{Cost of Owning} = \text{Cost of Renting} \tag{41}$$

$$\text{User Cost} = \text{Annual Rent} \tag{42}$$

$$\$24,000 = R \tag{43}$$

Therefore, you should be indifferent between buying and renting if the annual rent is:

$$\boxed{R = \$24,000 \text{ per year}}$$

Decision Rule:

- If annual rent $< \$24,000$: Rent the house
- If annual rent $> \$24,000$: Buy the house
- If annual rent $= \$24,000$: Indifferent between buying and renting

Hint: This assumes no capital gains on the house. How would expected appreciation affect this calculation?

5. An economy has full-employment output of 9000, and government purchases are 2000. Desired consumption and desired investment are as follows:

Real Interest Rate (%)	Desired Consumption	Desired Investment
2	6100	1500
3	6000	1400
4	5900	1300
5	5800	1200
6	5700	1100

(a) Why do desired consumption and desired investment fall as the real interest rate rises?

Solution: Desired consumption falls as the real interest rate rises because the higher return to saving encourages households to save more and reduce current consumption. Desired investment declines because the cost of capital (user cost) increases with the real interest rate, making investment projects less attractive and reducing the desired capital stock.

(b) Find desired national saving for each value of the real interest rate.

Solution: National saving is given by

$$S_d = Y - C_d - G,$$

where $Y = 9000$ and $G = 2000$.
Thus,

$$S_d = 9000 - C_d - 2000 = 7000 - C_d.$$

Calculate for each interest rate:

$r(\%)$	C_d	I_d	$S_d = 7000 - C_d$
2	6100	1500	900
3	6000	1400	1000
4	5900	1300	1100
5	5800	1200	1200
6	5700	1100	1300

(c) If the goods market is in equilibrium, what are the values of the real interest rate, desired national saving, and desired investment? Show that both forms of the goods market equilibrium condition, Eqs. (4.7) and (4.8), are satisfied at the equilibrium. Assume output is fixed at full employment.

Solution: Goods market equilibrium requires

$$Y = C_d + I_d + G.$$

Calculating $C_d + I_d + G$ for each r:

$r(\%)$	C_d	I_d	$C_d + I_d + G = C_d + I_d + 2000$
2	6100	1500	9600
3	6000	1400	9400
4	5900	1300	9200
5	5800	1200	9000
6	5700	1100	8800

The only r where $Y = 9000 = C_d + I_d + G$ is $r = 5\%$.
From part (b), at $r = 5\%$:

$$S_d = 1200, \quad I_d = 1200,$$

satisfying saving-investment equality $S_d = I_d$.
Thus, equilibrium is at

$$r = 5\%, \quad S_d = I_d = 1200.$$

(d) Repeat part (c) for the case in which government purchases fall to 1600. Assume consumption C_d is unchanged at each r.

Solution: With $G = 1600$, saving increases by 400:

$$S_d = Y - C_d - G = 9000 - C_d - 1600 = 7400 - C_d.$$

$C_d + I_d + G$ decreases by 400 for each r:

$r(\%)$	C_d	I_d	$S_d = 7400 - C_d$	$C_d + I_d + G$
2	6100	1500	1300	$6100 + 1500 + 1600 = 9200$
3	6000	1400	1400	$6000 + 1400 + 1600 = 9000$
4	5900	1300	1500	$5900 + 1300 + 1600 = 8800$
5	5800	1200	1600	$5800 + 1200 + 1600 = 8600$
6	5700	1100	1700	$5700 + 1100 + 1600 = 8400$

Equilibrium $Y = 9000 = C_d + I_d + G$ occurs at $r = 3\%$.
Saving-investment equality is:

$$S_d = 1400 = I_d,$$

so equilibrium values are:

$$r = 3\%, \quad S_d = I_d = 1400.$$

7. Suppose that the economywide expected future marginal product of capital is

$$\text{MPK}_f = 20 - 0.02K,$$

where K is the future capital stock. The depreciation rate of capital is $d = 0.20$ per period.

The current capital stock is 900 units. The price of a unit of capital is 1 unit of output. Firms pay taxes equal to 15% of their output. The consumption function in the economy is

$$C = 100 + 0.5Y - 200r,$$

where C is consumption, Y is output, and r is the real interest rate. Government purchases equal 200, and full-employment output is 1000.

(a) Suppose that the real interest rate is 10% per period. What are the values of the tax-adjusted user cost of capital, the desired future capital stock, and the desired level of investment?

Solution: The tax-adjusted user cost of capital is

$$uc/(1-\tau) = \frac{(r+d)p_k}{1-\tau} = \frac{(0.1+0.2)\times 1}{1-0.15} = 0.35.$$

Setting this equal to the future marginal product of capital,

$$20 - 0.02K = 0.35 \implies 0.02K = 19.65 \implies K = 982.5.$$

The desired investment is

$$I = K_{t+1} - K_t + dK_t = 982.5 - 900 + 0.2 \times 900 = 262.5.$$

(b) Now consider the real interest rate determined by goods market equilibrium. Now consider the real interest rate determined by goods market equilibrium. This part of the problem will guide you to this interest rate.

(a) Write the tax-adjusted user cost of capital as a function of the real interest rate r. Also write the desired future capital stock and desired investment as functions of r.

Solution: The tax-adjusted user cost of capital as a function of r is

$$uc/(1-t) = \frac{(r+0.2)\times 1}{1-0.15} = 0.235 + 1.176r.$$

Equate this to the marginal product of capital,

$$20-0.02K = 0.235+1.176r \implies 0.02K = 19.765-1.176r \implies K = 988.25-58.8r.$$

The investment function is

$$I = K - K_{-1} + dK_{-1} = (988.25 - 58.8r) - 900 + 0.2 \times 900 = 268.25 - 58.8r.$$

Solution: Goods market equilibrium requires

$$1000 = C + I + G = (100 + 0.5 \times 1000 - 200r) + (268.25 - 58.8r) + 200,$$

which simplifies to

$$1000 = 1068.25 - 258.8r.$$

Solving for r:

$$r = \frac{68.25}{258.8} \approx 0.264.$$

Substituting back to find equilibrium values:

$$C = 100 + 0.5 \times 1000 - 200 \times 0.264 = 547.2,$$

$$I = 268.25 - 58.8 \times 0.264 = 252.7,$$

$$S = 1000 - 547.2 - 200 = 252.8,$$

$$uc/(1-t) = 0.235 + 1.176 \times 0.264 = 0.545,$$

$$K = 988.25 - 58.8 \times 0.264 = 972.7.$$